

Introduction to Linear Algebra – Vectors

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Due Date

Introduction to Linear Algebra – Vectors

Linear algebra is a branch of mathematics that deals with vector spaces and their transformations. Vectors are essential mathematical entities used to represent quantities that have both magnitude and direction. Vectors form the foundation of linear algebra and play a crucial role in numerous scientific and engineering disciplines.

Definitions and Notation

Vectors

A vector is a mathematical object that represents both magnitude and direction. It is denoted by a boldface lowercase letter or an arrow above the letter, such as v or \vec{v} . In Cartesian coordinates, a vector v can be represented as $v = (v_1, v_2, \dots, v_n)$, where v_1, v_2, \dots, v_n are the components of the vector (Byjus, 2018).

Vector Notation

Let's consider a vector $v = (v_1, v_2, v_3)$ and its components. The component v_1 corresponds to the x-axis, v_2 to the y-axis, and v_3 to the z-axis. Thus, v can be written as $v = v_1i + v_2j + v_3k$, where $i, j, \text{ and } k$ are the unit vectors along the $x, y, \text{ and } z$ – axes, respectively (Beardon, 2008).

Vector Operations

Addition and Subtraction

Vector addition is performed by adding the corresponding components of two vectors. Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ be two vectors. The sum of u and v is given by $u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$. Vector subtraction is similar, and it is

performed by subtracting the corresponding components as follows

$$u - v = (u_1 - v_1, u_2 - v_2, u_3 - v_3).$$

Scalar Multiplication

Scalar multiplication involves multiplying a vector by a scalar quantity. Let $v = (v_1, v_2, v_3)$ be a vector, and c be a scalar. The scalar multiple of v is given by $cv = (cv_1, cv_2, cv_3)$.

Applications of Vectors

Vectors find applications in various fields, including physics, computer graphics, and engineering. Some common applications include (Strang & Herman, 2022):

1. Describing the motion of objects in physics and mechanics.
2. Representing forces, velocities, and accelerations in physics.
3. Solving systems of linear equations.
4. Modeling and simulating three-dimensional objects in computer graphics.
5. Analyzing electrical circuits and electromagnetic fields.

Vectors are versatile mathematical entities that are widely used in different scientific and engineering disciplines. Understanding vector properties, operations, and applications is essential for further studies in linear algebra. By mastering the concepts covered in this paper, one will develop a strong foundation in vector-based problem-solving and advance their mathematical skills.

References

- Beardon, T. (2008). An introduction to vectors. Retrieved from <https://nrich.maths.org/2390>
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- Strang, G., & Herman, E. J. (2022, September 7). 12: Vectors in space. Retrieved from [https://math.libretexts.org/Bookshelves/Calculus/Calculus_\(OpenStax\)/12%3A_Vectors_in_Space](https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/12%3A_Vectors_in_Space)

Appendix

Calculations and Formulas

Below are a few sample calculations and formulas involving vectors.

Vector Addition:

Given vectors $u = (2, 4, -1)$ and $v = (3, -2, 5)$, the sum of u and v is

$$u + v = (2 + 3, 4 + (-2), -1 + 5) = (5, 2, 4).$$

Scalar Multiplication:

Given vector $v = (1, -3, 2)$ and scalar $c = -2$, the scalar multiple of v is

$$cv = (-2 * 1, -2 * (-3), -2 * 2) = (-2, 6, -4).$$