Student Name

Instructor's Name

Course

Date

The Use of Integration to Determine the Volumes of Solids of Revolution: The Shell Method

General Background

Integration is one of the most useful methods in mathematics, particularly because of its countless applications. Most notably, the technique is used to compute volumes of solids, especially those created by revolving a curve around a specific axis (McGrath 109). The closed curve shown in Figure 1 below is rotated through 360 degrees about the y-axis, forming the solid of revolution depicted in Figure 2. To compute the volume of the solid, the shell method may be used, which is an integration technique for estimating the volumes of solids such as the one in Figure 2. The basic principle of the shell method is that given a curve bounded by x=b (upper limit) and x=a (lower limit), the volume of the solid formed by rotating the curve about the y-axis is given by:

$$V=2\pi\int_{a}^{b} r(x)\, h(x)\, dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where r(x) is the solid's radius, h(x) the height, and a and b the lower and upper limits, respectively.

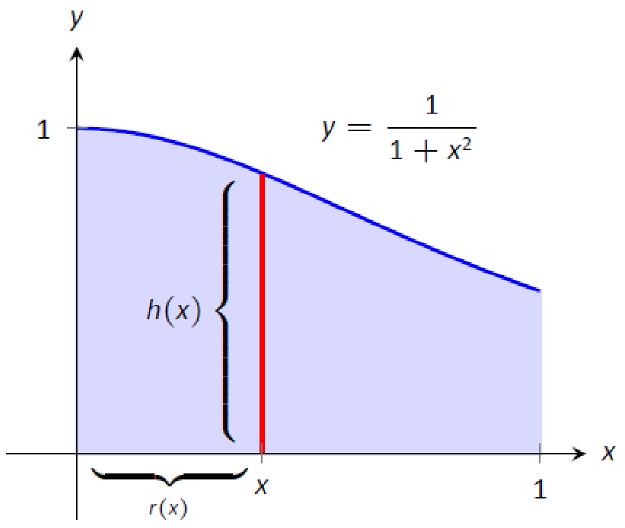

Figure 1. The Curve Rotated through 360 degrees about the Y-Axis (Hartman et al.)

Computing the Solid of Revolution Using the Shell Method

After rotating the curve, $y = \frac{1}{1+x^2}$, the solid depicted in Figure 2 is obtained.

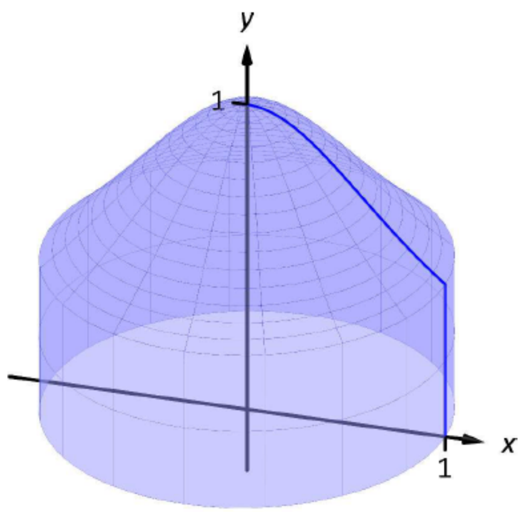

Figure 2. The Solid of Revolution Created by Revolving the Curve about the Y-Axis (Hartman et al.)

In Figures 1 and 2, the upper and lower limits are 1 and 0, respectively. Therefore, the solid's volume is given below:

V=2π$\int_a^b r(x)\ h(x)\ dx$; r(x)=x, while h(x)=y

V=2π$\int_0^1 x * \left(\frac{1}{1+x^2}\right)dx$

= 2π$\int_0^1 \left(\frac{x}{1+x^2}\right)dx$

To integrate by substitution,

let u=$1 + x^2$, then du=2x.dx and du/2=x.dx

The bounds become: u(0)= $1 + 0^2$ =1 and u(1)= $1 + 1^2$ =2

V=2π$\int_0^1 (\frac{x}{1+x^2})dx$

= 2π $* \frac{1}{2} \int_1^2 (\frac{1}{u})du$

= π $\int_1^2 \left(\frac{1}{u}\right)du$

= π ln(u)|2,1 = π[ln (2) – ln (1)]

= π[ln (2) – 0] = π*ln (2) = πln (2) units ^{3} or 2.178 units^3

Therefore, the solid of revolution obtained by rotating the curve, $y = \frac{1}{1+x^2}$, about the y-axis is πln(2) units ^{3} or 2.178 units^3.

Works Cited

Hartman et al. "The Shell Method." *LibreTexts*, Apex, https://math.libretexts.org/Bookshelves/Calculus/Calculus_3e_(Apex)/07%3A_Applications_of_Integration/7.03%3A_The_Shell_Method. Accessed 2023, May 25.

McGrath, Peter. "Newton's Shell Theorem via Archimedes's Hat Box and Single-Variable Calculus." *The College Mathematics Journal, 49*(2), 2018, pp.109-113.